Accretion of anisotropically diffusing particles

1984 J. Phys. A: Math. Gen. 17 L151

(http://iopscience.iop.org/0305-4470/17/3/010)

J. Phys. A: Math. Gen. **17** (1984) L151–L154. Printed in Great Britain

LETTER TO THE EDITOR

Accretion of anisotropically diffusing particles

T A Witten Jr

Exxon Research and Engineering, Annandale, New Jersey 08801, USA

Received 6 December 1983

Abstract. The structures formed in diffusion-controlled aggregation are modified if the diffusion constant of the aggregating particles becomes an anisotropic tensor. We relate this process to the accretion of isotropically diffusing oriented needles. We argue that anisotropy does not modify the Hausdorff dimension characterising the spatial structure of the aggregate.

When a cluster forms by the irreversible accretion of diffusing particles, it takes on a characteristic tenuous structure. This structure has a spatial scale invariance described by a Hausdorff dimension (Witten and Sander 1983, Meakin 1982). Recently, several variants of this process have been studied, involving variable sticking probability (Meakin 1982), increased concentration of diffusers (Nauenberg *et al* 1983, Witten and Meakin 1983), increased mean free path (Meakin 1983a, Bensimon *et al* 1983, Redner 1983), and the accretion of the clusters themselves (Meakin 1983b, Kolb *et al* 1983). In this note we consider the effect of an anisotropic diffusion constant. Such anisotropy would occur e.g. in the diffusion of atoms intercalated in layered compounds, or of heat in such compounds. We find that accretion by anisotropic diffusion is equivalent to accretion of anisotropic particles with a fixed orientation in space. We argue that this latter anisotropy only affects the structure of the aggregates at short distance scales, and that it does not affect the Hausdorff dimension at large scales. For diffusion reduced by a factor a^2 in one direction the overall shape of the aggregate is distorted by a factor a, but locally the structure remains isotropic.

We consider accretion as it occurs in the model of Witten and Sander. In this model, the cluster forms on a lattice, starting from a seed cluster which is the single site at the origin. The accreting particles are represented by a single random walker, which makes nearest-neighbour steps on the same lattice. Here, let the random walker make steps which are much smaller than the cluster lattice. This does not affect the structure of the cluster. To implement the anisotropic diffusion, we make the step length in the z direction different from that in the other directions. The diffusion constant in a given direction is proportional to the square of the step length in that direction. We consider the case where steps along the z axis are reduced in length by a factor a.

This model is clearly unchanged if the z dimension is uniformly dilated or compressed. This would be the effect, for example, of looking at the accretion process through a cylindrical lens. Such a distortion performed on the originally isotropic model would distort the cluster from its roughly spherical shape into an ellipsoidal shape. But it would not affect the dependence of the density–density correlation

0305-4470/84/030151+03$02.25

function on distance, even if this were isotropically averaged. Thus the distortion leaves the Hausdorff dimension unchanged.

For our anisotropically diffusing model, we may restore isotropic diffusion by stretching the z dimension by a factor a. This restores the step length in the z direction to parity with the other directions. However, it elongates the aggregated particles by the same factor. The anisotropic diffusion model is thus equivalent to the aggregation of isotropically diffusing oriented needles. At early stages of growth, these needles would certainly not have the Hausdorff dimension of isotropic accretion. Instead, the cluster would look roughly like a composite needle, and would have a Hausdorff dimension of one. But as growth proceeds, this shape is modified. Since it is nearly a line, the early cluster does not screen the diffusing field effectively. The diffusing flux onto an absorbing needle is only logarithmically dependent on its position. (The argument is the same as for hydrodynamic screening of rods. See Landau 1959). We neglect this variation and assume that accretion of a needle along the cluster is equally likely everywhere. From this assumption we may estimate how the shape of the needle evolves. At intermediate stages of the growth the width r of the cluster, as well as its length L, will be many lattice spacings. In general it may have an average density $\rho(r)$ characteristic of a fractal, i.e. $\rho(r) = r^{-c}$, with c a positive exponent. We may estimate the change in dimensions of the cluster when one needle is added, if we assume that the cluster is a cylinder of radius r and length L much greater than r. A needle is added to the end caps of this cylinder with a probability p which scales as the end cap area over the total:

$$p \sim r^{d-1}/(Lr^{d-2}) = r/L.$$

Such a needle adds a 'mass' a to the end and thereby makes an average change ΔL in the length of order

$$\Delta L r^{d-1} \sim a(r/L)\rho(r).$$

On the other hand a needle will attach to the curved walls of the cylinder with a probability of order unity. This increases the radius by an amount Δr given by

$$\Delta r r^{d-2} L \sim a\rho(r).$$

Thus both r and L change by the same order:

$$\Delta r \sim \Delta L \sim a\rho(r)/(Lr^{d-2}).$$

Since r and L increase by comparable amounts, the cluster becomes roughly spherical once r has grown to several times the initial length L_0. From then on the cluster remains spherical and the growth is indistinguishable from ordinary Witten–Sander growth except at length scales of order L_0 or smaller.

We may see the form of the anisotropic-diffusion cluster by compressing the needle cluster by a factor a, thus undoing the stretch transformation made above. The cluster now has an oblate shape, with aspect ratio a. Thus anisotropic diffusion distorts the overall shape of the cluster. Diffusion which is retarded in one direction produces a cluster flattened in that direction. The aspect ratio of the cluster is the square root of the anisotropy of the diffusion constant. As noted above the spatial correlations of the cluster have a form unaffected by a stretch or compression, so its fractal structure is the same as that of isotropic diffusion-limited aggregation. This kind of anisotropy of diffusion is thus irrelevant in the renormalisation-group sense.

This work benefited from discussions with Philip Pincus and Paul Chaikin.

References

Bensimon D, Domany E and Aharony A 1983 *Phys. Rev. Lett.* **51** 1394
Kolb M, Botet R and Julien R 1983 *Phys. Rev. Lett.* **51** 1123
Landau L D and Lifshitz I M 1959 *Fluid Mechanics* (London: Pergamon)
Meakin P 1982 *Phys. Rev.* A **26** 647
—— 1983a *Effects of Particle Drift on Diffusion-Limited Aggregation, DuPont preprint*
—— 1983b *Phys. Rev. Lett.* **51** 1119
Nauenberg M, Richter R and Sander L M 1983 *Phys. Rev.* B **28** 1649
Redner S 1983 Private communication
Witten T A and Sander L M 1983 *Phys. Rev.* B **27** 5686
Witten T A and Meakin P 1983 *Phys Rev.* A **28** in press